Orientational order in two dimensions from competing interactions at different scales

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We discuss orientational order in two dimensions in the context of systems with competing isotropic interactions at different scales. We consider an extension of the Brazovskii model for stripe phases including explicitly quartic terms with nematic symmetry in the energy. We show that leading fluctuations of the mean-field nematic solution drive the isotropic-nematic transition into the Kosterlitz-Thouless universality class; i.e., these systems have a thermodynamic phase with orientational quasi-long-range order.

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I. INTRODUCTION

Systems with competing interactions at different length scales are common in nature.[1] Examples go from highly correlated quantum systems such as quantum Hall samples[2] and high T_c superconductors[3] to classical systems such as ferromagnetic ultrathin films,[4,5] diblock copolymers,[6,7] colloidal suspensions,[8] ferromagnetic garnet films,[9] and magnetic fluids.[10] The essential phenomenology of these kinds of systems was described in a classic paper by Brazovskii.[11] Competition on different scales gives rise to ordered phases dominated by a nonzero wave vector in reciprocal space, as opposed to the usual $k=0$ long-range order. The nonzero value k_0 of the dominant wave vector gives rise to spatially modulated structures. In three dimensions Brazovskii[11] showed that striped phases appear through a first-order phase transition induced by fluctuations. Stripe patterns show both positional (anisotropic) and orientational long-range order, although the stripe solutions in the self-consistent Hartree approximation are marginally stable in three dimensions. Subsequent works extended the original results to two-dimensional systems in spite of the fact that, strictly speaking, fluctuations prevent any long-range order.[12,13]

Motivated by recent experimental observations of phases with modulated order in two-dimensional systems, we analyzed in a recent letter[14] the conditions for the existence of a purely orientational phase, a nematic phase, in models of the Brazovskii class. In the framework of the renormalization group (RG), we showed that an isotropic-nematic phase transition is generically present in these kinds of models, provided suitable quartic interactions between the basic degrees of freedom are taken into account. These interactions are naturally generated in the renormalization process. Furthermore, we found that, in two dimensions, renormalization of the Brazovskii model gives rise to an infinite number of relevant terms. We have shown in Ref. 14 that all those terms possess a common symmetry under rotations by π, a nematic symmetry. Keeping only the term with the highest symmetry, corresponding to quadrupole-quadrupole interactions, we showed that an isotropic-nematic phase transition is present and that it is of second order at mean-field level. Nevertheless, it was anticipated that the nature of the transition would probably be affected upon inclusion of fluctuations,[14] since it is not possible to break a continuous symmetry in two dimensions with short-ranged interactions.[15]

The original Brazovskii model in three dimensions is at its lower critical dimension, with fluctuations in the stripe solutions diverging logarithmically with the linear size of the system. The situation is more delicate in two dimensions where fluctuations are linearly divergent.[12] Then, if some order of this kind survives in two dimensions it must be purely orientational. However, it is still necessary to check whether orientational order survives to fluctuations of the relevant order parameter. Of course, in real systems, the nematic or even the stripe phase can be stabilized by other factors, such as anisotropies coming from the lattice substrate, the presence of impurities or some disorder that pin the stripe order. These effects will not be considered here, where we rest at the level of a completely isotropic system.

In the present work we pursue the analysis of a generalized Brazovskii model which takes into account quadrupolar interactions in two dimensions. We briefly review the mean-field treatment of Ref. 14, and we evaluate thermal fluctuations. We show that the isotropic-nematic transition belongs to the well-known Kosterlitz-Thouless universality class,[16] i.e., upon inclusion of order parameter fluctuations, the mean-field solution with nematic long-range order in fact retains only quasi-long-range orientational order. In turn, the stripe solution is unstable to fluctuations and a possible smecticlike phase reduces to a point at zero temperature. Similar results were found a long time ago by Toner and Nelson[17] in the context of defect mediated melting in two dimensions. In fact, both approaches are complementary and consistently lead to the same phase diagram. Emergence of algebraic order in O(n) models of the Brazovskii class has also been analyzed in a different context by Nussionv.[18] The present results were briefly anticipated by us in a reply[19] to a comment[20] to Ref. 14.

In the following, we introduce in Sec. II a prototypical model of the Landau-Ginzburg type for a nematic order parameter. We show that, while the mean-field treatment leads to a second-order phase transition in two dimensions, low energy fluctuations diverge logarithmically, as in the XY model of magnetism, destroying the long-range order and leading to an algebraic decay of the correlations. In Sec. III we introduce the extended Brazovskii model considered in Ref. 14 and briefly discuss the mean-field solution. We show that the relevant leading order fluctuations can be mapped to an XY type model. We thus show that, within the Gaussian approximation, the isotropic-nematic phase transition is in the Kosterlitz-Thouless universality class. The analysis also allows us to express the effective elastic constant $K(T)$ as a function of the parameters of the original model. A brief discussion of the results is given in Sec. IV.

II. LANDAU-GINZBURG THEORY FOR THE NEMATIC TRANSITION

The Landau-Ginzburg theory for a three-dimensional nematic phase is well known.[21] Here, we briefly restate it for a two-dimensional system because it presents some characteristics exclusive of the dimensionality of the problem. This analysis will be also helpful as a guide to the evaluation of fluctuations of the specific model of Sec. III.

A. Order parameter

The order parameter for a $2d$ nematic has two components to identify an orientation in the plane and the intensity, but not a direction. This means that it must be symmetric under the transformation $\theta \to \theta + \pi$.

Consider a complex number written in the form

$$Q = \alpha e^{i2\theta}, \tag{1}$$

where θ is an angle in $2d$ space. Then, if $\langle Q \rangle \neq 0$, the phase is said to have orientational order in the θ direction with the nematic symmetry $\theta \to \theta + \pi$. The nematic symmetry implies that the order parameter is not a vector. Instead, we can arrange the real and imaginary parts of Eq. (1) in a second rank symmetric and traceless tensor in the following way:

$$\mathrm{Re}(Q) = \alpha \cos(2\theta) \equiv Q_{xx} = -Q_{yy},$$

$$\mathrm{Im}(Q) = \alpha \sin(2\theta) \equiv Q_{xy} = Q_{yx}, \tag{2}$$

and

$$\hat{Q} = \begin{pmatrix} Q_{xx} & Q_{xy} \\ Q_{xy} & -Q_{xx} \end{pmatrix} = \alpha \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}. \tag{3}$$

Defining now a unit vector (the director) $\hat{n}$ with components $n_x = \cos\theta$ and $n_y = \sin\theta$, the nematic order parameter reads

$$\hat{Q} = \alpha \begin{pmatrix} n_x^2 - n_y^2 & 2n_x n_y \\ 2n_x n_y & n_y^2 - n_x^2 \end{pmatrix}, \tag{4}$$

or in component notation:

$$\hat{Q}_{ij} = 2\alpha \left(n_i n_j - \frac{1}{2} n^2 \delta_{ij} \right). \tag{5}$$

Equations (1) and (5) are two different ways of writing the same thing. We can now develop a Landau-Ginzburg free energy for a constant tensor near the transition, where $\langle Q \rangle$ is very small. The leading rotational invariant terms are

$$F(\hat{Q}) = \frac{1}{4} a_2 \mathrm{Tr}(\hat{Q}^2) + \frac{1}{8} a_4 \mathrm{Tr}(\hat{Q}^4) + \cdots. \tag{6}$$

Using Eq. (3) it is very simple to show that the free energy reduces to

$$F(\alpha) = \frac{1}{2} a_2 \alpha^2 + \frac{1}{4} a_4 \alpha^4 + \cdots. \tag{7}$$

Note that at this level the free energy is independent of θ, which means that it is invariant under arbitrary global rotations. In particular, in two dimensions, $\mathrm{Tr}(\hat{Q}^3)=0$ and therefore there are no terms with α^3, at variance with the $3d$ case. This implies that the mean-field isotropic-nematic transition is of second order in $2d$.

B. Mean-field phase transition

Consider the free energy of Eq. (7) and suppose that $a_4 > 0$. Therefore, if $a_2 > 0$ the only minimum of this energy is $\alpha=0$ and then $\langle Q \rangle = 0$. Conversely, if $a_2 < 0$ the minimum is at $\alpha = (-a_2/a_4)^{1/2}$ and

$$\langle Q \rangle = \sqrt{\frac{-a_2}{a_4}} e^{i2\theta}, \tag{8}$$

or in terms of the director components,

$$\langle \hat{Q}_{ij} \rangle = 2\sqrt{\frac{-a_2}{a_4}} \left(n_i n_j - \frac{1}{2} n^2 \delta_{ij} \right). \tag{9}$$

To leading order (near the transition) $a_2(T) = a(T - T^*)$, where $a > 0$ is a constant and T^* is the critical temperature. At the critical point the rotational symmetry in the plane is spontaneously broken. Choosing the director direction to correspond to $\theta=0$ then

$$\langle Q \rangle = \begin{cases} 0 & \text{if } T > T^* \\ \sqrt{\dfrac{a}{a_4}} (T^* - T)^{1/2} & \text{if } T < T^*. \end{cases} \tag{10}$$

This is the classic Landau-Ginzburg scenario for a second-order phase transition with $T_c = T^*$. We will see that fluctuations in the director orientation change this picture.

C. Fluctuations

We have developed the free energy of Eq. (7) considering that the order parameter Q is constant. However, if we want to study local fluctuations we can consider a local order parameter of the form $Q \equiv Q(x)$ and study the free energy for small variations of $Q(x)$ around the mean-field value Q. In order to do this we need to introduce terms proportional to derivatives of the order parameter in the expansion of the free energy. To leading order, we consider just first derivatives of Q and write a rotational invariant free energy of the form

$$F(\hat{Q}) = \frac{1}{V}\int d^2x\left\{\frac{\rho}{4}\mathrm{Tr}(\hat{Q}\hat{D}\hat{Q}) + \frac{1}{4}a_2\,\mathrm{Tr}(\hat{Q}^2) + \frac{1}{8}a_4\,\mathrm{Tr}(\hat{Q}^4) + \dots\right\}, \quad (11)$$

where ρ is a stiffness constant and the symmetric derivative tensor $\hat{D}_{ij} \equiv \nabla_i\nabla_j$.

Because the free energy is symmetric under global rotations, low energy angle fluctuations are the most relevant modes that rule the behavior of the system. We will see that in $2d$ the angular correlations are logarithmically divergent, ruling out true long-range order but showing instead quasi-long-range order or power-law decay of spatial correlations. Consider a local order parameter of the form

$$Q(x) = \sqrt{\frac{-a_2}{a_4}}e^{i2\theta(x)} \quad (12)$$

or in tensor form, as a function of the director components:

$$\hat{Q}_{ij}(x) = 2\sqrt{\frac{-a_2}{a_4}}\left[n_i(x)n_j(x) - \frac{1}{2}n^2\delta_{ij}\right]. \quad (13)$$

Thus, fixing the modulus to its mean-field value, we proceed to study small local fluctuations in the direction of the director in the nematic phase. Replacing Eq. (12) or Eq. (13) into Eq. (11) we find that

$$\delta F \equiv F[Q(x)] - F(\langle Q\rangle) = K(T)\int d^2x|\vec{\nabla}\theta(x)|^2, \quad (14)$$

where

$$K(T) = \frac{2|a_2|}{a_4}\rho. \quad (15)$$

Therefore, the free energy for the small angle fluctuations of the director can be mapped into the free energy of the *XY* model.[16] Angle correlations in the *XY* model decay algebraically as

$$\langle\cos[\theta(x) - \theta(0)]\rangle \propto x^{-\eta} \quad (16)$$

with $\eta = T/2\pi\rho$. Then, the isotropic-nematic transition in $2d$ belongs to the Kosterlitz-Thouless universality class with a disordering mechanism mediated by the unbinding of topological defects.[16] The only difference is that the role of vortices in the *XY* model is played here by disclinations.[17]

These results are independent of any microscopic mechanism. In the present case, if we begin with a Brazovskii-type Hamiltonian, one should be able to reach Eq. (11) where the parameters a_2, a_4, ρ, and T^* should be written in terms of the more "microscopic" ones.[14] This is the subject of the next sections.

III. MODEL WITH COMPETING ISOTROPIC INTERACTIONS AT DIFFERENT SCALES

A long time ago Brazovskii[11] introduced a rather general model with the aim of capturing the physics of systems with isotropic competing interactions at different scales. The model should be relevant for a wide class of systems as discussed in Sec. I. Specializing to two spatial dimensions and considering a scalar order parameter (Ising symmetry), the Brazovskii model is defined (in reciprocal space) by a coarse-grained Hamiltonian of the type

$$H_0 = \int_\Lambda \frac{d^2k}{(2\pi)^2}\phi(\vec{k})\left[r_0 + \frac{1}{m}(k-k_0)^2 + \dots\right]\phi(-\vec{k}), \quad (17)$$

where $r_0(T) \sim a(T-T_c)$, $k=|\vec{k}|$ and $k_0=|\vec{k}_0|$ is a constant given by the nature of the competing interactions. $\int_\Lambda d^2k \equiv \int_0^{2\pi}d\theta\int_{k_0-\Lambda}^{k_0+\Lambda}dk\, k$ and $\Lambda \sim \sqrt{mr_0}$ is a cutoff where the expansion of the free energy up to quadratic order in the momentum makes sense. The "mass" m measures the curvature of the dispersion relation around the minimum k_0 and the ellipsis in Eq. (17) indicates higher order terms in $(k-k_0)$. The correlator has a maximum at $k=k_0$ with a correlation length $\xi \sim 1/\sqrt{mr_0}$. Therefore, near criticality ($r_0 \to 0$), the physics is dominated by an annulus in momentum space with momenta $k \sim k_0$ and width 2Λ. This implies that at high temperatures the model possesses a continuous symmetry in momentum space, or in other words, a large phase space for fluctuations. The original model proposed by Brazovskii contains also an interaction term proportional to ϕ^4. In the mean-field approximation, this model leads to a second-order phase transition from an isotropic phase at high temperatures to an anisotropic stripe phase with modulation of the order parameter in the form

$$\langle\phi(x)\rangle = A\cos(k_0x). \quad (18)$$

Working in three dimensions, Brazovskii showed that including fluctuations of the order parameter self-consistently leads to a "fluctuation-induced first-order transition." Subsequently this transition was observed and studied in diblock copolymers.[22] In two dimensions, stripe phases arising from competing interactions are also observed in many systems, a notable example being that of $2d$ ultrathin ferromagnetic films with perpendicular anisotropy, in which the short-range exchange interaction between spins is frustrated by the long-range character of the dipolar interaction, giving rise to the well-known magnetic domains.[23] In recent years there have been indications that a mechanism similar to that proposed by Brazovskii can be at work in these systems.[24] However, stripe solutions are not stable with rigorously isotropic interactions, and fluctuations in the stripe direction diverge logarithmically in $3d$. Then, for two-dimensional systems the situation should be worse unless some isotropy-breaking effect be at work, like, e.g., lattice effects.[20] Nevertheless, even if positional long-range order is forbidden for such models in $2d$, one can ask if some kind of orientational order, reminiscent of stripe order, may survive in an isotropic model of the kind considered.

Recently,[14] we analyzed which kind of interaction terms could give rise to *purely* orientational order in two dimensions, besides the already known Brazovskii stripe solutions, which possess orientational as well as translational long-range order. We have considered a Hamiltonian of the type

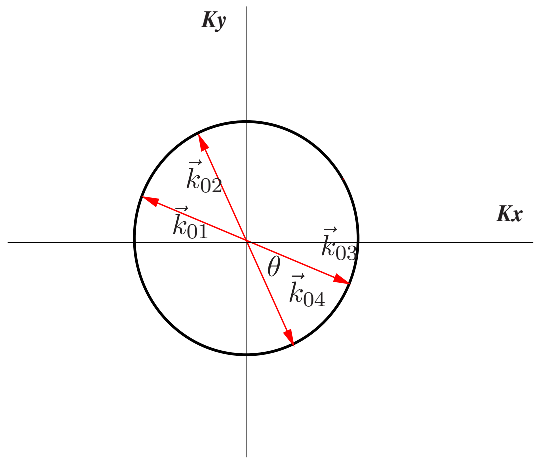

FIG. 1. (Color online) Representation of the four wave vectors of the interaction energy term reflecting the constraints imposed by symmetry in the Brazovskii model.

$$H_{\rm int} = \int_\Lambda \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_3}{(2\pi)^2}\frac{d^2k_4}{(2\pi)^2} u(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)\phi \times(\vec{k}_1)\phi(\vec{k}_2)\phi(\vec{k}_3)\phi(\vec{k}_4)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4), \quad (19)$$

where $u(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)$ is a smooth function of four momenta, and Λ indicates that the integrals should be taken in a circular shell $|\vec{k}-\vec{k}_0|<\Lambda$. At long distances, only low energy excitations matter, and $|\vec{k}|\sim|\vec{k}_0|$. Technically, we reduce the cutoff Λ and integrate over high energy degrees of freedom getting an effective low energy Hamiltonian following the usual Wilson renormalization-group procedure. Therefore, iterating this procedure, we implement the $\Lambda\to 0$ limit in which the interaction function u depends, in principle, on four angles (one for each momenta). However, this limit is strongly constrained by momentum conservation on the shell, and on global rotational invariance. As a consequence, the function $u(\vec{k}_{01},\vec{k}_{02},\vec{k}_{03},\vec{k}_{04})$ will depend solely on one angle θ as shown in Fig. 1.

In addition, permutation symmetry of the four momenta together with rotation invariance and momentum conservation force the relation $u(\theta+\pi)=u(\theta)$ that we recognize as the nematic symmetry. Taking this analysis into account,[14] we parametrize the interaction function as

$$u(\theta) = u_0 + u_2\cos(2\theta) + u_4\cos(4\theta) + \cdots, \quad (20)$$

where u_{2n} with $n=0,1,2\ldots$ are coupling constants characterizing different angular momentum channels of the interaction. Notice that this behavior is very different from the usual model without competing interaction where $k_0=0$. In the latter case only u_0 appears, avoiding any kind of spontaneous anisotropy in these systems.

The first term u_0 in Eq. (20) leads to the usual ϕ^4 theory considered by Brazovskii in his model for the isotropic-stripe transition. The other terms are all relevant in the RG sense.[14] At this point it is fair to say that the renormalization of theories with competing interactions at different scales is an open problem. In fact, the RG flux of the different couplings was not computed due to technical as well as conceptual difficulties. In any case, from this analysis we learn that it is not possible to ignore higher order momentum interactions when we deal with competing interactions. To simplify matters, we then proceeded to analyze the effect of the first of those terms, proportional to $\cos(2\theta)$. This is the first nontrivial interaction that leads to the isotropic-nematic phase transition. We do not expect that the other terms can qualitatively change our results on the isotropic-nematic phase transition. However, any higher order instability could drive the system to more complex anisotropic phases, such as tetragonal $[\cos(4\theta)]$ or hexatic $[\cos(6\theta)]$.

From the definitions in Sec. II A, we realize that the $\cos(2\theta)$ factor can be conveniently expressed in terms of the tensor order parameter and the interaction energy can be written in the form

$$H_{\rm int} = \int d^2x\{u_0\phi^4(\vec{x}) + u_2\,{\rm tr}\,\hat{Q}^2 + \gamma\,{\rm tr}\,\hat{Q}^4\} \quad (21)$$

with $\gamma>0$ and

$$\hat{Q}_{ij}(\vec{x}) = \phi(\vec{x})\left(\nabla_i\nabla_j - \frac{1}{2}\nabla^2\delta_{ij}\right)\phi(\vec{x}). \quad (22)$$

The gradients are related to the director $\hat{n}_i=\nabla_i/|\nabla_i|$. From Eq. (22) it is clear that the nematic order parameter is essentially a quadrupolar moment.

Next, we proceed to analyze this extended model in the self-consistent Hartree approximation.

A. Hartree approximation

Replacing in Eq. (21) $\phi^4\to\phi^2\langle\phi^2\rangle$ and ${\rm tr}\,\hat{Q}^2 \to {\rm tr}\{\phi(\nabla_i\nabla_j-\frac{1}{2}\nabla^2\delta_{ij})\phi\}\langle\hat{Q}_{ij}(\vec{x})\rangle$, where the mean values have to be determined self-consistently, we obtain a quadratic Hamiltonian in the Hartree approximation, which in reciprocal space reads

$$H_{\rm Hartree} = \frac{1}{2}\int\frac{d^2k}{(2\pi)^2}\phi(\vec{k})[\beta^{-1}C^{-1}(\vec{k})]\phi(-\vec{k}), \quad (23)$$

with the two-point correlator $C(\vec{k})$ given by[14]

$$C(\vec{k}) = \frac{T}{r+\frac{1}{m}(k-k_0)^2 - \alpha k^2\cos(2\theta)(u_2+\gamma\alpha^2)}. \quad (24)$$

Here

$$r = r_0 + u_0\int\frac{d^2k}{(2\pi)^2}C(\vec{k}) \quad (25)$$

and

$$\alpha = -\frac{1}{2}\int\frac{d^2k}{(2\pi)^2}k^2\cos(2\theta)C(\vec{k}), \quad (26)$$

where we have chosen $\langle Q_{ij}\rangle=2\alpha(n_in_j-\frac{1}{2}n^2\delta_{ij})$. θ is the angle subtended by $\vec{k}$ with the director $\hat{n}$.

Equations (24)–(26) must be solved self-consistently. Its solution has been discussed in Ref. 14. The main result which comes out is that in the case of attractive quadrupole interactions, $u_2<0$, Eq. (26) has nontrivial solutions for the nematic order parameter. Writing the equations in terms of adimensional parameters $r\to\tilde{r}T$, $k_0\to\tilde{k}_0\sqrt{mT}$ and $r_0\to\tau=a(1-T_c/T)$, one finds out that for high temperatures, $T>T_c$, the only possible solution is $\alpha=0$. Nevertheless, at $T=T_c$ a nematic phase emerges continuously with $\alpha\sim c(1-T/T_c)^{1/2}$ and the critical temperature $T_c=\frac{2}{m\tilde{k}^2}\sqrt{\frac{u_0}{|u_2|}}$.

A spontaneously broken continuous symmetry in a two-dimensional system with short-range isotropic interactions is forbidden by the Mermin-Wagner theorem.[15,18] In these systems, fluctuations of the order parameter typically diverge and the precise nature of the divergence can say if order is lost exponentially fast or if it decays more slowly giving rise to what is called "quasi-long-range-order." In order to analyze the effect of fluctuations of the nematic order parameter near the transition, we write the free energy of the model in the Hartree approximation. The partition function is

$$Z=\int \mathcal{D}\phi e^{-\beta H_{\rm Hartree}}=e^{-\beta F_H}. \tag{27}$$

Integrating over ϕ one arrives at

$$F_H=\frac{1}{2\beta}\mathrm{Tr}\ln C^{-1}. \tag{28}$$

In the limit $(k-k_0)^2/m\ll r_c$ (that is very near the Hartree critical temperature), the free energy reads

$$F_H=\frac{1}{2\beta}\mathrm{Tr}\ln\left\{\beta\left(r_c+\frac{(k-k_0)^2}{2m}-\bar{\alpha}k^2u_2\cos(2\theta)\right)\right\}, \tag{29}$$

where r_c and $\bar{\alpha}$ are the solutions of the self-consistent Hartree Eqs. (25) and (26) described in Ref. 14.

Since near the transition $\bar{\alpha}k_0^2u_2/r_c\ll 1$, we can expand the free energy in the form

$$F_H\approx\frac{1}{2\beta}\mathrm{Tr}\ln\left\{1-\frac{\bar{\alpha}k^2u_2}{r_c}\cos(2\theta)\right\}$$
$$\sim-\frac{u_2\bar{\alpha}}{2\beta r_c}\mathrm{Tr}\{k^2\cos(2\theta)\}-\frac{u_2^2\bar{\alpha}^2}{4\beta r_c^2}\mathrm{Tr}\{k^4\cos^2(2\theta)\}, \tag{30}$$

where a k-independent term was absorbed in F_H. The first term of the expansion is zero by symmetry (upon integration over θ), therefore at leading order the free energy reduces to

$$F_H=-\frac{u_2^2\bar{\alpha}^2}{4\beta r_c^2}\mathrm{Tr}\{k^4\cos^2[2(\theta-\varphi)]\}. \tag{31}$$

This expression represents the contribution of the anisotropic (nematic) part in the Hartree approximation, very near the transition into the nematic phase ($\bar{\alpha}\neq 0$). In the last expression we have introduced the angle φ that is the reference from which we measure the angle θ (which is the integration variable). At this level φ is an arbitrary constant, as it should be in a spontaneous symmetry breaking scenario. Next we will study smooth fluctuation of this field.

B. Fluctuations

In the same way we have done in the Landau-Ginzburg theory (and for the same reasons), we consider angle fluctuations of the order parameter, $Q(x)=\bar{\alpha}\exp i2\varphi(x)$. Therefore, the free energy now takes the form

$$F_H=-\frac{u_2^2\bar{\alpha}^2}{4\beta r_c^2}\mathrm{Tr}\{k^4\cos^2 2[\theta-\varphi(x)]\}. \tag{32}$$

The difficulty with this expression is the evaluation of the trace, since its argument is not diagonal in k nor in x space. To evaluate it, we make a coarse graining of configuration space, in such a way that in a small region around a point x_0 we consider φ essentially constant. Then, we can average over all points x_0 covering all configuration space. This coarse grained free energy can be diagonalized in k space and the trace can be easily evaluated. Consider the following expansion for $\varphi(x)$, for a fixed point x_0:

$$\varphi(x)=\varphi(x_0)+\vec{\nabla}\varphi(x_0)\cdot(\vec{x}-\vec{x}_0)+\ \dots\ \approx\varphi'(x_0)+\vec{\nabla}\varphi(x_0)\cdot\vec{x}, \tag{33}$$

where the constant $\varphi'(x_0)=\varphi(x_0)-\vec{\nabla}\varphi(x_0)\cdot\vec{x}_0$. With this expansion we rewrite the cosine in the expression for the free energy:

$$\cos 2(\theta-\varphi)\approx\cos 2[\theta-\varphi_0'-\vec{\nabla}\varphi(x_0)\cdot\vec{x}]$$
$$\approx\cos 2\theta'+2[\vec{\nabla}\varphi(x_0)\cdot\vec{x}]\sin 2\theta', \tag{34}$$

where $\theta'=\theta-\varphi_0'$ and we have considered smooth fluctuations $|\vec{\nabla}\varphi(x_0)|\ll 1$. Therefore,

$$\cos^2 2(\theta-\varphi)\approx\cos^2 2\theta'+4\cos 2\theta'\sin 2\theta'\vec{\nabla}\varphi(x_0)\cdot\vec{x}$$
$$+4[\vec{\nabla}\varphi(x_0)\cdot\vec{x}]^2\sin^2 2\theta'. \tag{35}$$

The first term contributes with an additive constant to the free energy, and then we will not consider it anymore. The second term is identically zero by symmetry considerations as shown in the Appendix. The relevant leading contribution to the fluctuations is the last one. Thus, let us consider the coarse grained free energy for smooth fluctuations:

$$F_{\rm fl}=-\frac{\bar{\alpha}^2u_2^2}{\beta r_c^2}\int\frac{d^2x_0}{V}\mathrm{Tr}\{k^4[\vec{\nabla}\varphi(x_0)\cdot\vec{x}]^2\sin^2 2\theta'\}, \tag{36}$$

where V is the volume of the system. Using the representation $\vec{x}=i\vec{\nabla}_k$, we write the trace in k space in the form

$$F_{\rm fl}=\frac{\bar{\alpha}^2u_2^2}{\beta r_c^2}\int d^2x_0\int\frac{dk}{(2\pi)^2}kd\theta\sin^2 2\theta[\vec{\nabla}\varphi(x_0)\cdot\vec{\nabla}_k]^2k^4. \tag{37}$$

The k derivatives can be evaluated as

$$[\vec{\nabla}\varphi(x_0)\cdot\vec{\nabla}_k]^2k^4=4|\vec{\nabla}\varphi_0|^2k^2\{1+2\sin^2\theta\}, \tag{38}$$

where $\vec{\nabla}\varphi_0\cdot\vec{k}=|\vec{\nabla}\varphi_0||\vec{k}|\sin\theta$ because $\vec{\nabla}\varphi_0$ is in the direction of the fluctuations (perpendicular to the director) and then the angle between $\vec{\nabla}\varphi_0$ and $\vec{k}$ is $\pi/2-\theta$.

We finally obtain

$$F_{\text{fl}} = \frac{\bar{\alpha}^2 u_2^2 \Gamma}{\beta r_c^2} \int d^2x_0 |\vec{\nabla}\varphi(x_0)|^2, \tag{39}$$

where

$$\Gamma = 4\int_{k_0-\Lambda}^{k_0+\Lambda} \frac{dk}{(2\pi)^2} k^3 \int_0^{2\pi} d\theta \sin^2(2\theta)\{1 + 2\sin^2\theta\} = \frac{1}{\pi}k_0^3\Lambda \tag{40}$$

to leading order in the cutoff $\Lambda \sim \sqrt{mr_0} = \sqrt{ma(T_c - T)}$.

Remembering that the Hartree solution of the order parameter is $\bar{\alpha} \sim c(1-T/T_c)^{1/2}$, we can write the free energy for the fluctuations as

$$F_{\text{fl}} = K(T) \int d^2x |\vec{\nabla}\varphi(x)|^2, \tag{41}$$

where the elastic constant is given by

$$K(T) = \kappa\left(1 - \frac{T}{T_c}\right)^{3/2}. \tag{42}$$

The constant $\kappa = \frac{128a^{1/2}}{15\pi}\frac{T_c}{\tilde{k}_0}$. Expression (41) is equal to Eq. (14). Then, Gaussian fluctuations of the nematic order parameter around the mean-field solution diverge logarithmically, and the nematic phase does not have true long-range order, but instead retains quasi-long-range order with the well-known Kosterlitz-Thouless phenomenology.[16] The conclusion is that fluctuations change the nature of the phase transition and, in particular, the critical temperature departs from its mean-field value T_c. The isotropic-nematic phase transition in the present model takes place at a temperature T_{KT}. At this temperature, a continuous phase transition mediated by unbinding of disclinations happens with $T_{\text{KT}} = (\pi/8)K(T_{\text{KT}})$ (Ref. 17). This relation, together with Eq. (42), allows to obtain the transition temperature as

$$T_{\text{KT}} = \frac{\pi\kappa}{8}\left(1 - \frac{T_{\text{KT}}}{T_c}\right)^{3/2}. \tag{43}$$

IV. CONCLUSIONS

Systems with competing interactions can develop complex ordered phases, with characteristics different from the usual ferromagnetic long-range order. For many systems, competition may lead to ground states with modulations in the order parameter. These broken symmetry states naturally show orientational order, and sometimes also positional one. While positional long-range order is strongly suppressed in two dimensions for systems with isotropic interactions and continuous symmetry, orientational order is more robust. We have studied a rather general model for competing interactions at different scales, looking for conditions for the existence of a purely orientational phase at low temperatures.

We have shown that, in the two-dimensional Brazovskii model, the quartic interactions with higher derivatives of the order parameter are all relevant terms in the renormalization-group sense. All these terms can be arranged and interpreted as representing multipole interactions. Among them, the quadrupole-quadrupole interaction is the first nontrivial contribution. A mean-field solution of the model gives rise to an isotropic-nematic phase transition. The analysis of Gaussian fluctuations around the mean-field solution leads to a phase diagram similar to the one found by Toner and Nelson[17] in the context of defect mediated melting in two dimensions. Positional order of Brazovskii stripe solutions is destroyed by thermal fluctuations, which are known to diverge linearly in two dimensions. However, orientational quasi-long-ranged order is preserved in the nematic phase of the extended model. We have shown that there is a critical temperature T_{KT} at which orientational quasi-long-range order is destroyed. By analogy with the *XY* model, one can conclude that the disordering of the nematic phase takes place by means of a disclination unbinding mechanism, and the isotropic-nematic phase transition is in the Kosterlitz-Thouless universality class.

The main difference of our approach with that of the Toner-Nelson-Kosterlitz-Thouless is that our model allows for an analysis of both sides of the phase transition. This fact makes possible to characterize the transition in terms of "microscopic" parameters, which describe the underlying competing interactions. Also, within the present formalism, it is possible to alternatively interpret the nematic phase as a quadrupole condensation rather than a melting of topological defects.

Finally, the presence of a nematic phase from competing interactions in two dimensions can be present in a variety of systems such as ultrathin ferromagnetic films with perpendicular anisotropy,[25] block copolymers,[26] microemulsions and colloids,[8,27] between others. The detection and quantitative characterization of such phases in those systems rely on novel imaging techniques which are at present rapidly evolving.

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APPENDIX

We show here that the second term in Eq. (35) gives zero contribution to the free energy. The contribution of this term to the free energy is

$$F_2 \sim 4i\int d^2k \cos 2\theta' \sin 2\theta' \vec{\nabla}\varphi(x_0) \cdot \vec{\nabla}_k k^4 \tag{A1}$$

with

$$\vec{\nabla}\varphi(x_0)\cdot\vec{\nabla}_k k^4 = 4k^2\vec{\nabla}\varphi(x_0)\cdot\vec{k} = k^3|\vec{\nabla}\varphi(x_0)|\sin\theta. \quad (A2)$$

The last expression is due to the fact that if we measure θ from the director and consider that $\vec{\nabla}\varphi(x_0)$ is orthogonal to it, then the angle between $\vec{\nabla}\varphi(x_0)$ and $\vec{k}$ is $\pi/2-\theta$. Then the scalar product is written in terms of $\cos(\pi/2-\theta)=\sin\theta$. Introducing in Eq. (A1) and remembering that $\theta'=\theta-\varphi_0'$,

$$F_2 \sim 16i|\vec{\nabla}\varphi(x_0)|\int dk k^4\int_0^{2\pi} d\theta \cos 2(\theta-\varphi_0') \times\sin 2(\theta-\varphi_0')\sin\theta. \quad (A3)$$

The angular integral is identically zero whatever the value of φ_0', thus $F_2=0$.

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